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# AN ADAPTIVE METHOD OF TEST SELECTION IN SYSTEM DEVELOPMENT

N. H. Hakansson

PREPARED FOR:
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1700 MAIN ST. • SANTA MONICA • CALIFORNIA • 90406

## PREFACE

Experience indicates that the major sources of uncertainty in weapon systems development can be traced to those activities which involve testing and redesign. Yet, surprisingly little of a conceptual nature has been done to improve the decisionmaking process involved in performing these activities. In this paper, an adaptive model of the development process is presented which enables decisionmakers to determine which tests, if any, should be performed at a given stage and what corrective action, if any, should be taken once the test results are known.

This study should be useful to the System Project offices, the Air Force Research and Technology Division, the Air Staff, and the directors of various test facilities. It should also be of interest to the Office of the Secretary of Defense and to defense contractors, as well as to persons concerned with commercial development.

## SUMMARY

This Memorandum considers the problem of selecting "optimal" or "best" programs for the conduct of testing and redesign activities in the development of military hardware. An adaptive model of the testing process is presented which is designed to (1) provide the project director and his staff with a means for determining the best test to perform at a given stage in the development of a system, and (2) enable the same decisionmakers to choose intelligently between the available redesign actions once the test results are known. While the model is presented in terms of relatively simple systems and tests, it is capable of handling systems and tests of a highly complex nature.

The final checkout phase of a new system is considered first. The model developed by Johnson [5] for the purpose of determining the optimal trouble-shooting procedure when an operating system fails is found to be an appropriate abstraction of this phase and is adopted as a starting point. This model is then generalized to include the case in which the object tested is destroyed when the test is unsuccessful. In addition, sufficiency conditions for the optimality of the test procedure suggested by Johnson are given.

In Sec. III the problem of test selection in the early stages of development is superimposed on the final checkout phase. It is well known that the imperfect nature of the information gained in early testing tends to offset the advantages of early redesign. A dynamic programming formulation is given which enables one to determine which test, if any, should be performed at a given stage and what corrective action, if any, should be taken once the test result is known. As a

by-product, a means for determining the net benefit of a given test at a given stage is obtained. The use of the multistage decision model developed in this section is illustrated by means of a comprehensive example.

The situation in which there is serial dependence among certain test information messages, as in the case of a multistage rocket, is examined in Sec. IV. In the last section, it is shown that Nelson's model of parallel R&D efforts [6] may be viewed as a special case of the model developed in this paper.

## ACKNOWLEDGMENT

The author wishes to acknowledge a debt to F. S. Pardee of The RAND Corporation who suggested the need for this study and commented extensively on the draft of the manuscript.

## CONTENTS



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# I. INTRODUCTION

The aim of this study is to establish a framework and a methodology for selecting "optimal" or "best" programs for the conduct of tests in the development of military hardware. For expository purposes, the end product of the development process may be considered to be a new weapon system of some complexity, such as a missile, a fighter, or a radar system. The hardware to be developed, which will be called a system, will be viewed as consisting of several subsystems (e.g., propulsion, guidance). Each subsystem, in turn, is considered to be made up of several components; each component may consist of parts, etc. Systems whose subsystems are indivisible will be referred to as one-level systems; systems whose subsystems are at most made up of indivisible components will be called two-level systems, etc.

Development testing is generally regarded as distinct from and preceding production or operational testing. Its purpose is to determine whether a particular new system design will work (i.e., satisfy given performance requirements, including endurance) in the environment in which it is intended to function. Development testing is concluded as soon as a prototype design is found to operate satisfactorily in this environment. Production testing in the usual sense is then initiated to determine, and perhaps to improve, the reliability of the production process instituted to manufacture the system in quantity. In the present study, we concentrate exclusively on development testing.

## BASIC CHARACTERISTICS OF THE DEVELOPMENT ENVIRONMENT

Development in the area of defense takes place in an environment which, although highly difficult to cope with, can easily be described.

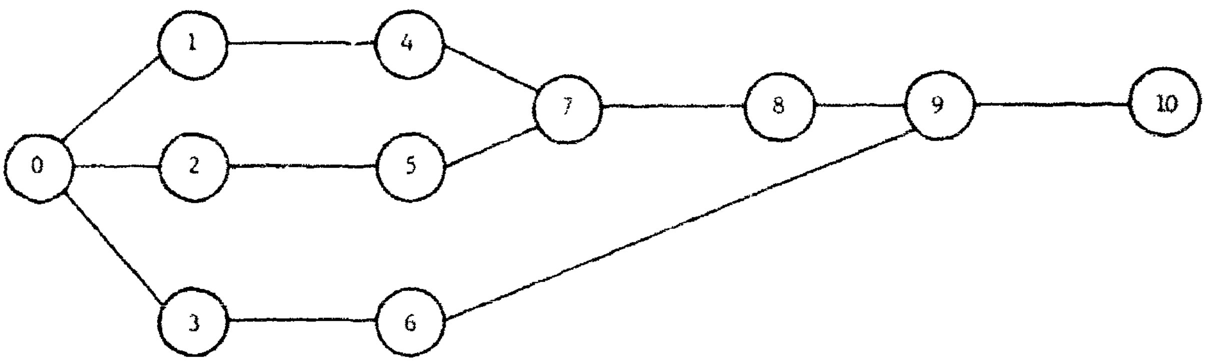
Thus, there is general agreement that the chief characteristics of the environment faced by the developer are uncertainty, the necessity of making decisions based on current knowledge, and the generation of new knowledge as the development proceeds. Each of these characteristics is formally incorporated into the models constructed in this study via the approach of modern decision theory.

## SETTING OF THE PROBLEM

The problem to be discussed is that faced by the project director and those of his subordinates and contractors who must decide what tests to perform, if any, at each stage in the development of a system. A closely related decision problem concerns the extent of redesign, if any, to be done once the results of a given test are known. While the project director and the test supervisors concerned must, by reference to a flexible or conditional listing of possibilities, consider the whole future of the project in deciding what to do next at a given point in the development, they cannot specify the precise sequence of steps to be taken for the whole project in advance. This is because they are in a situation which is adaptive: the best action to take at a given point is a function of the test results obtained so far.

Accordingly, the models in this paper will be developed with two purposes in mind. First, to provide the project director and his staff with a means for determining the "best" test to perform at a given stage in the development. Second, to enable the same decision-makers to choose intelligently among available actions once the test results are known.

Consider the abstraction given in Fig. 1 of a typical (but highly




Key

0 Begin development
1 Complete construction of component 1
2 Complete construction of component 2
3 Complete construction of component 3
4 Complete testing and redesign of component 1
5 Complete testing and redesign of component 2
6 Complete testing and redesign of component 3
7 Complete assembly of subsystem 1
8 Complete testing and redesign of subsystem 1
9 Complete assembly of system
10 Complete testing and redesign of system

Fig. 1 - PERT Network for a System Consisting of Three Components

simplified) development project. Experience indicates that the major sources of uncertainty in development can be traced to those activities that involve testing and redesign (i.e., those that end in events 4, 5, 6, 8, and 10). These activities, then, should logically be the prime targets in an attempt to provide a basis for improved decision-making in the development field, and will be so viewed in this paper.

## OBJECTIVE FUNCTION

The objective of development is to bring to "completion" a new system or item not previously available. "Completion" may be charac-

terized by a triplet of measurements: money cost x, time elapsed t, and system performance r. It will be assumed that the developer[†] has preferences over the various triplets (x, t, r) that the system could exhibit once it is "completed." These preferences will be represented by the function $U(x, t, r)$ which is defined for all non-negative values of x, t, and r. It has the property that $U(x_1, t_1, r_1) > U(x_2, t_2, r_2)$ if and only if the triplet $(x_1, t_1, r_1)$ is preferred to the triplet $(x_2, t_2, r_2)$. Presumably, U is decreasing in x, nonincreasing in t, and nondecreasing in r.

In most cases, the system performance element of the triplet would itself be a vector $r = (r_1, \ldots, r_n)$, the components of which would correspond to such magnitudes as speed, range, CEP, weight, reliability, etc. To simplify the exposition we assume that a given performance level $\bar{r}$ is to be achieved and that no additional utility is derived by exceeding $\bar{r}$. Thus,

$$U(x, t, r') = U(x, t, \bar{r})$$

for all x, t, and all $r'$ such that $r' \geq \bar{r}$, that is, $r'_i \geq \bar{r}_i$ for all i, and

$$U(x, t, r'') = U(x, t, 0)$$

for all x, t, and all $r''$ such that $r'' < \bar{r}$.

Let $V(x, t) = -U(x, t, \bar{r}) + K$, where K is a constant such that $V(0, 0) = 0$. Referring to t as the time cost, we may call V the time-and-money cost function.

V will now be specified more precisely. The utility of money in

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[†]The person or persons charged with making the basic decisions for a given system.

a large organization, such as the government, tends, as a result of the aggregation of many individuals' functions, to be linear. Also, a single development project usually requires only a small portion of the government's total resources, so that a nonlinear utility function would be well approximated by its tangent at some point in the relevant region. Thus we lose little in our representation of the real world and gain much by way of simplicity by assuming V to be linear in money.

Now it can easily be shown that this condition implies that V is decomposable and may be written

$$V(x, t) = x + a(t),$$

where, since V is nondecreasing in t, a is a nondecreasing function (not necessarily linear). While we would expect a to be linear for most projects (that is, the "cost" per time unit to be fixed), it is probably a sharply increasing function in the typical crash program. In this Memorandum, a(t) will be assumed to be identically zero to keep the discussion as simple as possible; in a later study we will consider both the money cost and the time cost.†

PROBABILITIES

The required performance level $\bar{r}$ of the system establishes a standard against which, at any point in time, the developer may subjectively derive a two-valued probability function in advance for each part, component, and subsystem. As the development proceeds, the developer

†There is no reason why the inverse procedure could not be chosen. The decision to consider the money cost rather than the time cost first was completely arbitrary.

will have numerous opportunities to modify his initial probabilities by conducting tests. The conditional distributions of the outcomes (messages) of these tests will be assumed to be known. Of course, in real world situations, these distributions would also have to be determined subjectively since in development there is, by definition, no directly applicable experience to fall back on.

We do not consider here the question of how to derive subjective probability distributions. Numerous descriptions of how this may be accomplished, on the basis of postulates presupposing certain consistencies in behavior, are available in the literature.†

Neither do we argue the merits of using subjective probabilities in relation to alternative inputs. We merely call attention to two relevant observations. As McKean [3] points out, a decisionmaker, in taking a position on an uncertain issue, <u>implicitly quantifies</u> considerations which he refuses to quantify explicitly. Going one step further, it follows as a theorem that a decisionmaker who observes a certain measure of consistency in deciding under uncertainty in fact <u>imputes</u> a probability distribution over the possible outcomes, regardless of what criterion is used. This distribution is such that if it is used to solve the decision problem under risk, it will give the same solution as was obtained under uncertainty with the given criterion.‡ As a result, if one is committed to this (highly reasonable) measure of consistency, it would seem that one might as well convert the decision problem to one under risk by searching for the necessary probability distribution(s).

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†See, for example, the accounts of Marschak [1] and Timson [2].

‡See Ref. 4, pp. 287-294.

## II. THE FINAL CHECKOUT PROBLEM WHEN TEST RESULTS ARE TWO-VALUED

This section deals with the problem of minimizing the m ney cost of a particular system once the final test stage has been reached.† It wil' be assumed that in testing a system, subsystem, component, or part the only information gained is whether it works or doesn't work. In other words, the test information is two-valued.

### ONE-COMPONENT SYSTEMS

To begin with, consider the case where a one-level (one-component) system consisting of M parts has reached the final test stage. The problem is how to conduct the final test and any redesign that may be required so as to maximize total utility. In this case, this is equivalent to minimizing the expected cost of the final test since all other costs have already been incurred.

Since the required performance level is given, the outcome of the final test is either "success" or "failure." The following assumptions will now be made:

Assumption 1. The final test of a component (subsystem, system) will fail if and only if one or more of its parts (components, subsystems) is not working.

Assumption 2. The failure of one part of a component is independent of the failure of any other part or combination of parts.

The test procedure given in Fig. 2 will be arbitrarily selected for this model. (Conditions under which this procedure is optimal are

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†The final test stage is considered first because in an adaptive situation the optimal decision rules, as will become evident shortly, are obtained by working backwards in time.

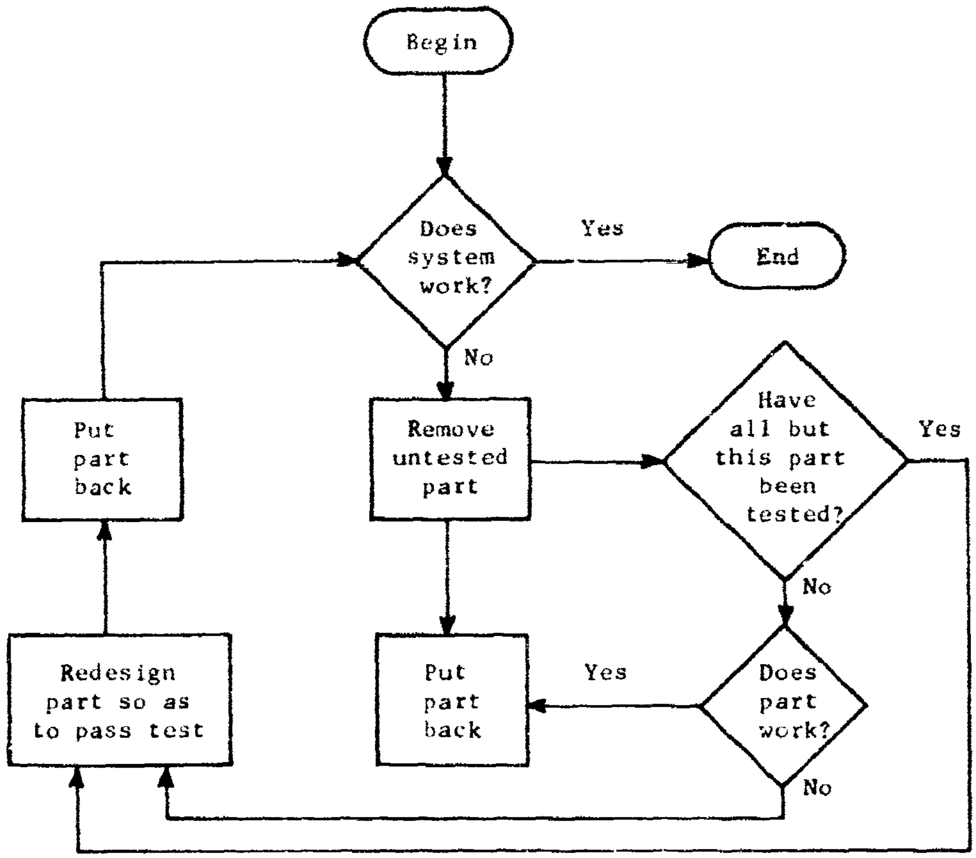



Fig. 2 - Flow Chart of Test Procedure for Final Checkout of One-component System

given later.) Having so limited the problem, the task becomes to select the sequence in which the parts should be tested in order to minimize expected cost. This problem was first treated by Johnson [5]; it will be briefly reviewed here in the setting of the current study.

The following definitions will be used:

$E_i$ = cost of testing component i

$M_i$ = number of parts in component i

$R_{ij}$ = cost of removing jth part from ith component

$E_{ij}$ = cost of testing jth part of ith component

$R'_{ij}$ = cost of returning part j back into component i

$p_{ij}$ = a priori probability that jth part of ith component works

$C_{ij}$ = cost of redesigning and testing the jth part of the ith component so as to pass jth part test, given that it failed this test the first time

$S_i$ = sequence in which parts of ith component are tested

$C_i(S_i)$ = total expected cost of final checkout of component i after initial component test when sequence $S_i$ is used

$B_{ij} = R_{ij} + C_{ij} + R'_{ij}$

$q_{ij} = 1 - p_{ij}$

$\pi_{ij} = \Pi_{k=j}^{M_i} p_{ik}$

We now make another assumption:

Assumption 3. In each test, $p_{ij} > 0$ for all i and j.

This assumption merely expresses the common-sense rule that we will not engage in any test unless the a priori probability that all parts involved will work is positive.

Given an arbitrary sequence $S_i$, we obtain, assuming that the system is not destroyed if the final test fails,

$$C_i(S_i) = \pi_{i1}0 + (1 - \pi_{i1})B_{i1} + q_{i1}(C_{i1} + E_i)$$

$$+ (1 - \pi_{i2})B_{i2} + q_{i2}(C_{i2} + E_i) + \cdots$$

$$+ (1 - \pi_{iM_i})B_{iM_i} + q_{iM_i}(C_{iM_i} + E_i) - q_{iM_i}E_{iM_i},$$

or

$$(1) \qquad C_i(S_i) = \sum_{j=1}^{M_i} [B_{ij} + q_{ij}(C_{ij} + E_i)] - \sum_{j=1}^{M_i} \pi_{ij}B_{ij} - q_{iM_i}E_{iM_i}.$$

After the $j - 1$th part has been checked out, the probability that the component test will fail again is $(1 - \pi_{ij})$. The last term of (1) indicates that we need not test the last part if the procedure takes us that far because then we would know that the part does not work and would immediately begin to redesign it.

For any given $\bar{p}_i = (p_{i1}, \ldots, p_{iM_i})$, it is clear that with reference to the terms in brackets, $C_i(S_i)$ is independent of the particular test sequence used. However, the values of the last two terms are clearly affected by the way the tests of the parts are sequenced.

Let us temporarily fix the final part in the test sequence and consider sequence $S_i'$, which is the same as $S_i$ except that parts $j$ and $j + 1 < M_i$ have been interchanged. This gives

$$\begin{aligned} C_i(S_i') - C_i(S_i) &= (1 - \pi_{ij})B_{ij} + q_{ij}(C_{ij} + E_i) \\ &\quad + (1 - \pi_{i,j+1})B_{i,j+1} + q_{i,j+1}(C_{i,j+1} + E_i) \\ &\quad - (1 - \pi_{ij})B_{i,j+1} - q_{i,j+1}(C_{i,j+1} + E_i) \\ &\quad - (1 - p_{ij}p_{i,j+2}, \ldots, p_{iM_i})B_{ij} - q_{ij}(C_{ij} + E_i) \\ &= -B_{ij}(\pi_{ij} - p_{ij}p_{i,j+2}, \ldots, p_{iM_i}) - B_{i,j+1}(\pi_{i,j+1} - \pi_{ij}) \\ &= -\pi_{i,j+2}p_{ij}B_{ij}(p_{i,j+1} - 1) - \pi_{i,j+1}B_{i,j+1}(1 - p_{ij}). \end{aligned}$$

Simplifying further,

$$(2) \qquad C_i(S_i') - C_i(S_i) = -\pi_{i,j+2}(-p_{ij}B_{ij}q_{i,j+1} + p_{i,j+1}B_{i,j+1}q_{ij}).$$

Clearly, this expression is negative if

$$-p_{ij}B_{ij}q_{i,j+1} + p_{i,j+1}B_{i,j+1}q_{ij} > 0;$$

that is, if

$$(3) \qquad \frac{p_{ij}B_{ij}}{q_{ij}} < \frac{p_{i,j+1}B_{i,j+1}}{q_{i,j+1}}.$$

As a result, if (3) holds, sequence $S_i'$ leads to a lower expected final checkout cost than $S_i$.

Thus, since (3) is a transitive relation over the values of j, we can, by successively interchanging adjacent parts until (3) holds for all $j < M_i - 1$, obtain the optimal sequence for any given final part. Formally, the rule may be stated as

$$(4) \qquad j \text{ precedes } k \text{ if } \quad \frac{p_{ij}B_{ij}}{q_{ij}} < \frac{p_{ik}B_{ik}}{q_{ik}}.$$

We must now consider the final part in the sequence. Let $S_{iM_i}$ be the sequence obtained by applying (4) to all parts $j = 1, \ldots, M_i$. Let $S_{ik}$ be $S_{iM_i}$ with the kth part transferred to the end. Thus the problem is to determine $m_i$ such that

$$(5) \qquad G_{im_i} = \min_k G_{ik},$$

where

$$\begin{aligned} G_{ik} &= C_i(S_{ik}) - C_i(S_{iM_i}) \\ &= (1 - \pi_{i,k+1}p_{ik})B_{i,k+1} + (1 - \pi_{i,k+2}p_{ik})B_{i,k+2} + \cdots \\ &\quad + (1 - \pi_{iM_i}p_{ik})B_{iM_i} + (1 - p_{ik})B_{ik} - q_{ik}E_{ik} \\ &\quad - [(1 - \pi_{ik})B_{ik} + (1 - \pi_{i,k+1})B_{i,k+1} + \cdots \\ &\quad + (1 - \pi_{iM_i})B_{iM_i} - q_{iM_i}E_{iM_i}] \end{aligned}$$

$$= -p_{ik}(B_{ik} + \pi_{i,k+1}B_{i,k+1} + \cdots + \pi_{iM_i}B_{iM_i}) + \sum_{j=k}^{M_i} \pi_{ij}B_{ij}$$

$$- q_{ik}E_{ik} + q_{iM_i}E_{iM_i}$$

$$= (1 - p_{ik}) \sum_{j=k+1}^{M_i} \pi_{ij}B_{ij} - p_{ik}B_{ik} + \pi_{ik}B_{ik} - q_{ik}E_{ik} + q_{iM_i}E_{iM_i}.$$

Simplifying,

$$(6) \qquad G_{ik} = q_{ik} \sum_{j=k+1}^{M_i} \pi_{ij}B_{ij} + B_{ik}(\pi_{ik} - p_{ik}) - q_{ik}E_{ik} + q_{iM_i}E_{iM_i}.$$

Thus, $G_{ik}$ can easily be generated, providing the desired minimizing index $m_i$.

Let $K_i$ denote the set of indices $k'$ for which $p_{ik'} = 1$. Then by (4), $K_i$ will be at the end of $S_i$, unless $m_i \notin K_i$ as determined by (5), in which case $K_i$ will appear immediately preceding the final part in $S_i$. In the first case, $1 - \pi_{ik'} = 1 - p_{ik'} = 0$ for all $k' \in K_i$, which implies by (1) that the testing would never reach the parts in $K_i$; letting $\bar{S}_i = S_i - K_i$, we obtain $C_i(\bar{S}_i) = C_i(S_i)$. Thus, in this case, it is immaterial whether we exclude $K_i$ from the test sequence.

In the second case, $1 - \pi_{ik'} = 1 - \pi_{im_i} = 1 - p_{im_i}$ so that

$$C_i(\bar{S}_i) = C_i(S_i) - (1 - p_{im_i}) \sum_{k' \in K_i} B_{ik'},$$

or

$$(7) \qquad C_i(\bar{S}_i) < C_i(S_i).$$

Since we clearly do not want to test parts that we know are working, we must therefore explicitly exclude them in defining the test sequence. This gives

Theorem 1. To ensure that parts known to be working are not tested for any component $i$, it is necessary to limit the domain of $S_i$ to those indices $j$ for which $p_{ij} < 1$, $j = 1, \ldots, M_i$.

The main result may now be stated as

Theorem 2. For any component $i$, number the parts for which $p_{ij} < 1$ in ascending order of $p_{ij}B_{ij}/q_{ij}$, $j = 1, \ldots, M_i$; then transfer the mth element of $S_i$ to the final position $M_i'$ where $m_i$ is given by $G_{im_i} = \min_k G_{ik}$, $k = 1, \ldots, M'$, and $G_{ik}$ is given by

$$G_{ik} = q_{ik} \sum_{j=k+1}^{M_i'} \pi_{ij}B_{ij} + B_{ik}(\pi_{ik} - p_{ik}) - q_{ik}E_{ik} + q_{iM_i'}E_{iM_i'}. \tag{8}$$

Then this is the optimal sequence.

Let us call the optimizing sequence $S_i^*$. Then

$$C_i(S_i^*) = \min_{S_i \in \mathcal{S}_i} C_i(S_i),$$

where $\mathcal{S}_i$ is the set of all possible sequences for component $i$. Clearly $S_i^*$ is a function of $\bar{p}_i$, the probability vector of the component, so that we should write $S_i^*(\bar{p}_i)$. It will be convenient to define $C_i^*(\bar{p}_i) \equiv C_i(S_i^*(\bar{p}_i))$; thus $C_i^*(\bar{p}_i)$ is the optimal expected checkout cost when the given test procedure is used and $\bar{p}_i$ holds.

## MULTI-COMPONENT SYSTEMS

We now consider two-level systems and find that the problem remains exactly the same, as shown by Johnson [5]. The test procedure is depicted

in Fig. 3. We use the convention of testing a given component even after the last part has been redesigned, although this is not strictly necessary.

Let $i$ denote the $i$th component of the system, using an arbitrary numbering scheme, where $i = 1, \ldots, K$.

We now need the following definitions:

$E$ = cost of performing system test

$R_i$ = cost of removing $i$th component from system

$R_i'$ = cost of returning working component $i$ into system

$p_i = \Pi_{j=1}^{M_i} p_{ij} = \pi_{i1}$ = a priori probability that $i$th component works

$S$ = sequence in which components are tested

$C(S)$ = total expected cost of final checkout, excluding initial test of system, when the components are tested in the sequence $S$

$B_i = R_i + E_i + R_i'$

$q_i = 1 - p_i$

$\pi_i = \Pi_{j=i}^{K} p_j$

We now obtain

$$C(S) = 0\pi_1 + (1 - \pi_1)B_1 + q_1\left(\frac{C_1^*(\bar{p}_1)}{q_1} + E\right) + (1 - \pi_2)B_2 + q_2\left(\frac{C_2^*(\bar{p}_2)}{q_2} + E\right)$$

$$+ \cdots + (1 - \pi_K)B_K + q_K\left(\frac{C_K^*(\bar{p}_K)}{q_K} + E\right) - q_K E_K,$$

or

$$C(S) = \sum_{i=1}^{K} (B_i + C_i^*(\bar{p}_i) + q_i E) - \sum_{i=1}^{K} \pi_i B_i - q_K E_K. \tag{9}$$

Since this is analogous to (1), we obtain, by the proof of Theorems 1 and 2, Theorems 3 and 4.

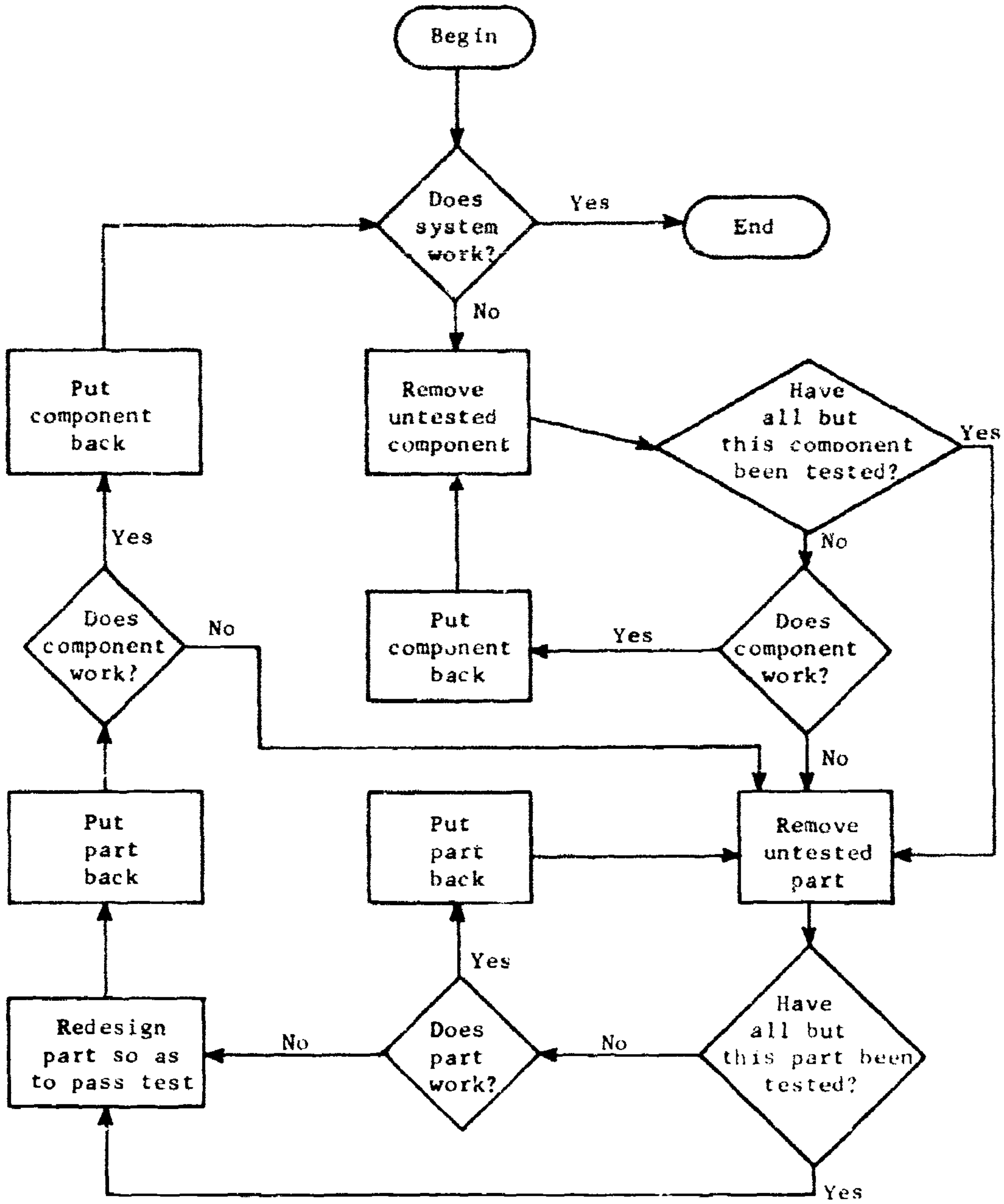



Fig. 3 – Flow Chart of Test Procedure for Final Checkout of Multi-component System

Theorem 3. To ensure that components known to be working are not tested, it is necessary to limit the domain of S to those indices i for which $p_i < 1$, $i = 1, \ldots, K$.

Theorem 4. Number the components for which $p_i < 1$ in ascending order of $p_i B_i / q_i$, $i = 1, \ldots, K$; then transfer the mth element of S to the final position $K'$ where m is given by

$$G_m = \min_k G_k, \qquad k = 1, \ldots, K'$$

and

$$G_k = q_k \sum_{i=k+1}^{K'} \pi_i B_i + B_k(\pi_k - p_k) - p_k E_k + p_{K'} E_{K'}$$

in analogy with (8). Then this is the optimal sequence $S^*$.

Let $\bar{p} = (\bar{p}_1, \ldots, \bar{p}_K)$ and let $C^*(\bar{p}) \equiv C(S^*(\bar{p}))$. Then the minimum expected final checkout cost, including the initial system test, becomes

$$f_j(\bar{p}) \equiv E + C^*(\bar{p}). \tag{10}$$

Clearly for $\bar{p} = I$, where I is the unit vector, $C^*(I) = 0$. Also, $C^*(\bar{p})$ appears to be continuous for $\bar{0} < \bar{p} < I$ (where $\bar{0}$ is the zero vector) with discontinuities at $p_{ij} = 1$ for all i and j, and strictly decreasing in $p_{ij}$ for all i, j.

It is readily seen that the preceding results generalize so as to apply to systems of any number of levels.

## LOSS OF SYSTEM

So far, we have assumed that failure in the final test of an item does not result in loss of the item. We now consider the situation

where this is the case; let the cost of a duplicate system be L and the cost of a duplicate of component i be $L_i$. Then

$$C(S) = \pi_1 0 + (1 - \pi_1)(L_1 + E_1 + R_1') + C_1^*(\bar{p}_1) + q_1(E + L)$$

$$+ (1 - \pi_2)(L_2 + E_2 + R_2') + C_2^*(\bar{p}_2) + q_2(E + L) + \cdots$$

$$+ (1 - \pi_{K'})(L_{K'} + E_{K'} + R_{K'}') + C_{K'}^*(\bar{p}_{K'})$$

$$+ q_{K'}(E + L) - p_{K'}E_{K'},$$

or

$$\text{(10a)} \qquad C(S) = \sum_{i=1}^{K'} (L_i + E_i + R_i' + C_i^*(\bar{p}_i) + q_i(E + L))$$

$$- \sum_{i=1}^{K'} \pi_i(L_i + E_i + R_i') - q_{K'}E_{K'}.$$

This is analogous to (9) with $R_i$ replaced by $L_i$, and E replaced by E + L. Thus Theorems 2 and 4 also hold when the object tested is destroyed in the final checkout as long as $R_i$ is replaced by $L_i$; that is, $B_i = L_i + E_i + R_i'$.

## THE TEST PROCEDURE: SUFFICIENCY CONDITIONS FOR OPTIMALITY

So far, we have taken the test procedures shown in Figs. 2 and 3 as given. Although chosen arbitrarily, as pointed out earlier, the procedure appears rather reasonable. We now give the conditions under which it is optimal. While these conditions will be stated for one-level systems only, they essentially "generalize" when multilevel systems are involved.

We first prove the following:

Lemma. Let

$$(11) \qquad \frac{p_i B_i}{1 - p_i} \le \frac{p_{i+1} B_{i+1}}{1 - p_{i+1}}, \qquad i = 1, \ldots, K' - 1,$$

where $E$, $B_i > 0$, $0 < p_i < 1$, $i = 1, \ldots, K'$. Then

$$(12) \qquad p_1\Big(B_1 \prod_{\substack{i=1 \\ i \ne j}}^{K'} p_i + E\Big) - E \ge 0, \qquad j = 1, \ldots, K'$$

implies that

$$(13) \qquad p_k\Big(B_k \prod_{\substack{i=1 \\ i \ne j}}^{K'} p_i + E\Big) - E \ge 0, \qquad j = 1, \ldots, K', \quad k = 2, \ldots, K'.$$

Proof. For any $k = 2, \ldots, K'$, (11) gives

$$(14) \qquad p_1 B_1 \le \frac{p_k B_k (1 - p_1)}{1 - p_k}.$$

The inequality (12) is then preserved if the right-hand side of (14) is substituted for $p_1 B_1$ in (12), giving

$$\frac{p_k B_k (1 - p_1)}{1 - p_k} \prod_{\substack{i=1 \\ i \ne j}}^{K'} p_i + p_1 E - E \ge 0, \qquad j = 1, \ldots, K',$$

which reduces to (13) upon factoring out $1 - p_1$.

Theorem 5. For the test procedure in Fig. 2 to be optimal, it is sufficient that

$$(12)\qquad p_1\Big(B_1 \prod_{\substack{i=1\\ i\neq j}}^{K'} p_i + E\Big) - E \geq 0, \qquad j = 1, \ldots, K',$$

where the subscript i denotes the ith part and the parts have been numbered in ascending order of $p_iB_i/(1 - p_i)$.

Proof. We first show that the procedure in Fig. 2 is optimal given that a system test is performed first, and then show that a system test should indeed be performed as the first step.

(1) Given that the initial system test fails, an alternative test procedure must necessarily encompass the testing and repairing of two or more parts before a new system test is made. Let Q(P) be the total checkout cost when parts s, ..., s′ are grouped together somewhere in the test sequence followed by parts s′ + 1, ..., K′ (separably or in groups). Similarly, let Q(P′) be the total checkout cost when $k \in \{s, \ldots, s'\}$ is "broken out" of the group; that is, after the other parts in the group (but before parts s′ + 1, ..., K′) have been tested, a system test is performed to determine whether further testing is needed. We then obtain

$$(15)\quad Q(P) - Q(P') = \cdots + (1 - p_s, \ldots, p_{K'})(B_s + \cdots + B_{s'})$$

$$+ (1 - p_s)C_s + \cdots + (1 - p_{s'})C_{s'}$$

$$+ (1 - p_s, \ldots, p_{s'})E + \cdots$$

$$- [\cdots + (1 - p_s, \ldots, p_{K'})(B_s + \cdots + B_{k-1}$$

$$+ B_{k+1} + \cdots + B_{s'}) + (1 - p_s)C_s + \cdots$$

$$+ (1 - p_{s'})C_{s'} + (1 - p_s, \ldots, p_{k-1}p_{k+1}, \ldots, p_{s'})E$$

$$+ (1 - p_k p_{s'+1}, \ldots, p_{K'})B_k + (1 - p_k)E]$$

$$= (1 - p_s, \ldots, p_{k-1}p_{k+1}, \ldots, p_{s'})$$

$$\cdot \left[p_k\left(B_k \prod_{i=s'+1}^{K'} p_i + E\right) - E\right].$$

Since the first factor of (15) is always positive and the second factor is non-negative whenever (12) is by the lemma, it follows that part k should be "broken out" whenever (12) holds, and, since k may be any part, that every part in every group should be "broken out." Consequently, the test procedure in Fig. 2 is optimal given that a system test is performed as the first step.

(2) Let $C(P_i)$ be the total final checkout cost when the test procedure in Fig. 2 is used, except that no system test is performed until after the ith part test. We then obtain

$$(16) \qquad C(P_1) - C(P_0) = B_1 + (1 - p_1)C_1 + E + \cdots$$

$$- [E + (1 - p_1, \ldots, p_{K'})B_1 + (1 - p_1)C_1$$

$$+ (1 - p_1)E + \cdots]$$

$$= p_1(p_2, \ldots, p_{K'}B_1 + E) - E,$$

which is non-negative whenever (12) is. Similarly,

$$C(P_i) - C(P_{i-1}) = P_i(P_{i+1}, \ldots, P_K, B_i + E) - E, \qquad (17)$$

$$i = 2, \ldots, K',$$

which, by the lemma, is non-negative whenever (12) is. Consequently, the theorem holds.

## III. MULTISTAGE TESTING

Since the minimum expected cost of the final checkout, $C^*(\bar{p})$, is a function of the *a priori* probabilities that the various parts are working, we are now in a position to assess the value of performing tests of parts, components, and subsystems at earlier stages in the development. Before constructing an analytical model to deal with this problem, several important aspects of its setting must be clarified.

We have already pointed out that each element of $\bar{p}$ expresses the *a priori* probability that a particular part will perform its job *in the final system*. Thus at any one point in time, the same part may have an entirely different *a priori* probability of working in one system as opposed to another.

In effect Assumption 1 states (perhaps somewhat idealistically) that the tests used for the final checkout provide perfect information. Letting z denote the state of the system and y the message of the test as to what that state is, perfect information implies the following conditional distribution P(y/z):

| | P(y/z) Message | | |
|---|---|---|---|
| State of System | Working $y_1$ | Not Working $y_2$ | $\Sigma_y$ P(y/z) |
| Working ($z_1$) | 1 | 0 | 1 |
| Not working ($z_2$) | 0 | 1 | 1 |

When a test is performed at an earlier stage in the development, we would not expect to obtain a conclusive answer as to whether the

part, component, or subsystem tested will work in the final system. Thus we might expect to have a situation like this:

| | P(y/z) Message | | |
|---|---|---|---|
| State of Part | $y_1$ | $y_2$ | $\Sigma_y$ P(y/z) |
| $z_1$ | 1 | 0 | 1 |
| $z_2$ | 1/4 | 3/4 | 1 |

In this case, $P(y_1/z_1) = 1$ and $P(y_2/z_1) = 0$. This implies that if the part is working the test will bear this out, and in most real situations this will probably be true. However, if we wish, for example, to take into account the possibility that the test equipment itself may be faulty without our discovering it, we obviously have a situation in which $P(y_2/z_1) > 0$.

In the example, $P(y_2/z_2) = 3/4$; that is, if the part is not working, the probability that the test will discover this fact is 3/4. $P(y_2/z_2)$ may be considered as a measure of the "power" of the test. In general, the earlier the stage in which the test is made, the smaller we would expect its "power" to be.

For a test to have any value, its results must provide learning, that is, for a modification of the _a priori_ probabilities involved. The mechanism by which this modification is accomplished is provided by Bayes' Theorem. Letting P(z) denote the _a priori_ probability of state z, we obtain by this theorem

$$P(z/y) = \frac{P(y/z)P(z)}{\Sigma_z \; P(y/z)P(z)}.$$

In the above example, letting $P(z_1) = 1/2$,

$$P(z_1/y_1) = \frac{P(y_1/z_1)P(z_1)}{P(y_1/z_1)P(z_1) + P(y_1/z_2)P(z_2)} = \frac{1 \cdot 1/2}{1 \cdot 1/2 + 1/4 \cdot 1/2} = 4/5,$$

$$P(z_2/y_1) = \frac{1/4 \cdot 1/2}{1 \cdot 1/2 + 1/4 \cdot 1/2} = 1/5,$$

$$P(z_1/y_2) = \frac{P(y_2/z_1)P(z_1)}{P(y_2/z_1)P(z_1) + P(y_2/z_2)P(z_2)} = 0,$$

$$P(z_2/y_2) = 1 - P(z_1/y_2) = 1.$$

Thus, on the basis of this test, we would revise the probability that the part is working from 1/2 to 4/5 if message $y_1$ is received, and to 0 if $y_2$ is obtained. Note that if $P(y/z) = x_y$, say, for all z, the a posteriori probabilities would coincide with the a priori probabilities. Therefore, for learning to take place, the test must generate messages that are in some way correlated with the relevant states of the part tested.

When $y_2$ is received, it will frequently seem logical to take some corrective action in the form of rework or redesign, since this would normally be cheaper than to do so when the part has already become part of a larger unit. However, the same action appears rather illogical when message $y_1$ is received in the test. On the other hand, the imperfect nature of the information gained in early testing tends to offset the advantages of early redesign. Thus, we have a "dual" problem: what test to perform on the given part and what corrective action to take once the result is known.

PRE-CHECKOUT TESTING OF PARTS

Let us make the following definitions:

$\bar{p}$ = *a priori* probability vector of system

$\bar{p}'$ = *a posteriori* probability vector of system

$y$ = a possible test result

$Y$ = the set of possible test results of an available test

$\mathcal{Y}$ = the set of available tests (includes null test, $Y_0$)

$a$ = a corrective action

$A$ = the set of corrective actions (includes null action, $a_0$)

$T(Y)$ = cost of performing test $Y$

$c_Y(y, a)$ = expected cost of taking corrective action a when message y is received and test Y is used

$y'(y, a)$ = transformed message $y'$ after action a has been taken following receipt of message y

Let us now discuss some properties of $c_Y$ and $y'$. Clearly, $c_Y(y, a_0) = 0$ for all y, and $y'(y, a_0) = y$ for all y since "no action" costs nothing and is incapable of performing a transformation of a message. Returning to our example in which $y_1$ is the message that the part works and $y_2$ that it does not, let $a_1$ be the action consisting of redesigning the part until it passes the test (message $y_1$ is obtained). We then obtain $y'(y_2, a_1) = y_1$. In general, the effect of taking a corrective action other than $a_0$ is to transform a "bad" message into a "good" one while at the same time incurring a cost.

We can now state the problem in which we have at some point an opportunity to test and redesign one part before the final checkout stage is reached:

$$\text{(18)} \qquad \min_{Y \in \mathcal{Y}} \left\{ T(Y) + \sum_{y} P(y) \min_{a \in A} \left[ c_Y(y, a) + C^*(\bar{p}'(\bar{p}, y'(y, a))) \right] \right\}.$$

As stated earlier, the problem is two-fold. First, for a given test and test result we must find the least-cost, or "best," action a. Then, weighting this cost by the probability of receiving the given message and summing over all messages, it remains, after repeating this procedure for all tests Y, to pick the test with the least expected cost. The "best" test may, as stated earlier, be the null test.

The function $\alpha^*_{Y,\bar{p}}$, among all possible $\alpha_{Y,\bar{p}}$, which associates with each possible $y \in Y$ the action $a \in A$ that minimizes the bracketed expression of (18) is the decisionmaker's best decision rule for that test and a priori probability vector $\bar{p}$. Letting $Y^*$ be the best test for the part that can be tested prior to the final checkout, the minimum expected test and redesign cost of the system is

$$(19) \qquad T(Y^*) + \sum_{y \in Y^*} P(y)[c_{Y^*}(y, \alpha^*_{Y^*,\bar{p}}(y)) + C^*(\bar{p}'(\bar{p}, y'(y, \alpha^*_{Y^*,\bar{p}}(y))))].$$

Since the minimum expected test and rework cost of the system when the part is not tested prior to the final test is $C^*(\bar{p})$, the value of the information gained by conducting test Y is clearly

$$(20) \qquad C^*(\bar{p}) - T(Y) - \sum_{y \in Y} P(y)[c_Y(y, \alpha^*_{Y,\bar{p}}(y)) + C^*(\bar{p}'(\bar{p}, y'(y, \alpha^*_{Y,\bar{p}}(y))))];$$

that is, the amount by which the minimum total expected cost of testing is reduced when test Y is employed.

PRE-CHECKOUT TESTING OF COMPONENTS

Just as opportunities exist to test parts before they are assembled into components, opportunities are usually available to test components before they are assembled into subsystems, to test subsystems before

assembly, etc. Here we need to draw on the preceding discussion of pre-checkout testing of parts as well as the discussion in Sec. II.

Let $p_i$ be the probability that the ith component works before the test (of the component) and $p_i'$ be the a posteriori probability. By Assumption 2,

$$p_i = \prod_{j=1}^{M_i} p_{ij}.$$

Once the result of the test is known, $p_{ij}'$ can now be obtained, as before, by Bayes' Theorem. Again by Assumption 2,

$$p_i' = \prod_{j=1}^{M_i} p_{ij}'.$$

In taking corrective action in response to a message y which relates to a component, we again assume that the procedure in Fig. 3 is used. Thus the cost of taking corrective action becomes a function of $\bar{p}_i$ as well as of the message y and the action a. The minimum expected such cost, $c_Y^*(y, a, \bar{p}_i)$, would be derived by the method of Sec. II.

The value of the information gained by conducting pre-checkout test Y of component i (when other components and parts are not so tested) is then, in analogy with (20),

$$(21) \qquad C^*(\bar{p}) - T(Y) - \sum_{y \in Y} P(y)[c_Y^*(y, \alpha^*_{Y,\bar{p}}(y), \bar{p}) + C^*(\bar{p}'(\bar{p}, y'(y, \alpha^*_{Y,\bar{p}}(y))))].$$

The minimum expected test and rework cost of the system now becomes

$$(22) \qquad \min_{Y \in Q_j} \left\{T(Y) + \sum_{y \in Y} P(y) \min_{a \in A} [c_Y^*(y, a, \bar{p}) + C^*(\bar{p}'(\bar{p}, y'(y, a)))]\right\}.$$

THE COMPLETE PROBLEM

Having formulated the test and rework problem for two levels, we are now in a position to state the entire development problem in sequential form. Consider Fig. 4 in which the test stages involved in the development of a two-level system are indicated. Any activity, or the smallest set of activities, which can be finished before all remaining activities must begin, constitutes a stage. Thus, a stage is the smallest set of activities during which information that can be used for subsequent stages can be obtained.

It is now clear that at any stage the choice of a test is often a choice of a combination of tests; we therefore use the word "test" to refer to a particular combination of tests (see, for example, stage 1 in Fig. 4).

We now make the following definitions:

$N$ = number of stages at which tests can be performed

$f_N(\bar{p})$ = minimum expected cost of developing system when tests exist at N stages

$Y_N$ = the set of possible test results of a test available at Nth stage from end

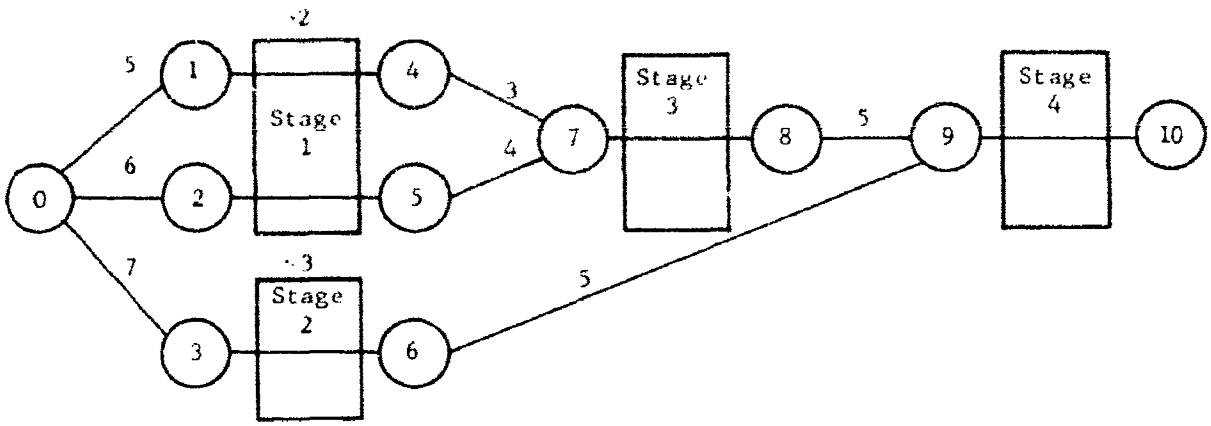
$\mathcal{Y}_N$ = the set of tests available at Nth stage from end (includes null test)

$A_N$ = the set of redesign actions available at Nth stage from end

$T(Y_N)$ = cost of performing test $Y_N$

$c^*_{Y_N}(y, a, \bar{p})$ = minimum expected cost of taking corrective action a when test $Y_N$ is used, test result y is received, and the a priori probability vector is $\bar{p}$

$K$ = cost of producing and assembling parts and components exclusive of testing and redesign activities




Key

0 Begin development
1 Complete construction of component 1
2 Complete construction of component 2
3 Complete construction of component 3
4 Complete testing and redesign of component 1
5 Complete testing and redesign of component 2
6 Complete testing and redesign of component 3
7 Complete assembly of subsystem 1
8 Complete testing and redesign of subsystem 1
9 Complete assembly of system
10 Complete testing and redesign of system

Fig. 4 - Test Stages of a System Consisting of Three Components

Extending our two-stage model to N stages, we obtain for the minimum expected cost of developing the system, by the principle of optimality [6, p. 83],

$$(23) \qquad f_N(\bar{p}) = K + \min_{Y_N \in \mathcal{Y}_N} \Big\{ T(Y_N) + \sum_{y \in Y_N} P(y) \min_{a \in A_N} [c^*_{Y_N}(y, a, \bar{p}) + f_{N-1}(\bar{p}'(\bar{p}, y'(y, a)))] \Big\},$$

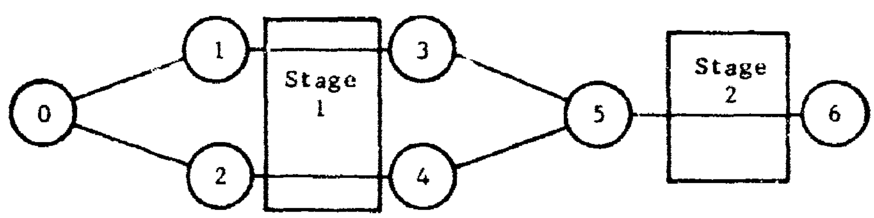
where $f_1(\bar{p}) = C^*(\bar{p})$.

At any stage, this equation enables the developer to choose the best test for that stage. (Remember that the best test may, of course, be no

test.) Since the best decision at any stage always depends on past actions and test results, the model is an adaptive one.

AN EXAMPLE

At this point, we provide an example to illustrate the use of the preceding model. Let the task be to develop, at minimum cost, a new missile system (system X), which is to meet a given performance level. For purposes of illustration, we assume that system X consists of only two subsystems (parts)--a guidance subsystem and a propulsion subsystem. The PERT chart governing its development is given in Fig. 5, where the two possible test stages are indicated.




Key

0 Begin development
1 Complete construction of guidance subsystem
2 Complete construction of propulsion subsystem
3 Complete testing and redesign of guidance subsystem
4 Complete testing and redesign of propulsion subsystem
5 Complete assembly of total system
6 Complete testing and redesign of total system

Fig. 5 - System X

The relevant information concerning the final (second) test stage is summarized in Table 1. Tables 2 through 9 and the text following contain the necessary information required for decisionmaking with respect to test stage 1.

Table 1

COSTS RELATING TO FINAL TEST STAGE

Missile System X:

| | |
|---|---|
| $E = 1$ | Expected cost of performing final test firing of one missile. |
| $L = 7$ | Expected cost of replacing a system used up in a final test firing. |

Guidance Subsystem (Subsystem 1):

| | |
|---|---|
| $E_1 = 10$ | Expected cost of tests required to be performed on guidance subsystem to ascertain whether it meets the required performance level (includes the production costs of guidance and auxiliary subsystems used up in these tests), given that the final system test failed. |
| $L_1 = 4$ | Expected cost of (replacing) guidance subsystem (used up in a final system test firing). |
| $R_1' = 1$ | Expected cost of integrating guidance subsystem with propulsion subsystem to form total system. |
| $C_1 = 8$ | Expected cost of redesigning and testing guidance subsystem to pass the guidance subsystem test, given that this test was not passed the first time (includes the cost of replacing guidance and auxiliary subsystems used up in this phase). |

Propulsion Subsystem (Subsystem 2):

| | |
|---|---|
| $E_2 = 12$ | Expected cost of tests required to be performed on propulsion subsystem to ascertain whether it meets the required performance level (includes the production costs of propulsion and auxiliary subsystems used up in these tests), given that the final system test failed. |
| $L_2 = 2$ | Expected cost of (replacing) propulsion subsystem (used up in a final system test firing). |
| $R_2' = R_1' = 1$ | |
| $C_2 = 6$ | Expected cost of redesigning and testing propulsion subsystem to pass the propulsion subsystem test, given that this test was not passed the first time (includes the cost of replacing propulsion and auxiliary subsystems used up in this phase). |

Table 2

TESTS AVAILABLE AT TEST STAGE 1

| Designation[a] | Configuration | | | | |
|---|---|---|---|---|---|
| | No Test | Centrifuge Test of Guidance Subsystem | Sled Test of Guidance Subsystem | Static Firing Test of Propulsion Subsystem | Cost T(Y) |
| $Y_0$ | X | | | | 0 |
| $Y_1$ | | X | | | .1 |
| $Y_2$ | | | X | | 2 |
| $Y_3$ | | | | X | 1 |
| $Y_{1,2}$ | | X | X | | 2.1 |
| $Y_{1,3}$ | | X | | X | 1.1 |
| $Y_{2,3}$ | | | X | X | 3 |
| $Y_{1,2,3}$ | | X | X | X | 3.1 |

[a]For simplicity, in the case of $Y_{1,2}$, we assume that $Y_1$ and $Y_2$ provide independent messages.

Table 3

PROPERTIES OF TEST $Y_1$

(Centrifuge Test of Guidance Subsystem)

| State of Subsystem with respect to System X Requirements | Probability of Receiving Message y Given State z (P(y/z)) | | |
|---|---|---|---|
| | Test Passed at High Speed ($y_1$) | Test Passed at Medium Speed Only ($y_2$) | Test Failed ($y_3$) |
| Working ($z_1$) | .8 | .2 | 0 |
| Not working ($z_2$) | .2 | .4 | .4 |

Table 4

PROPERTIES OF TEST $Y_2$

(Sled Test of Guidance Subsystem)

| State of Subsystem with respect to System X Requirements | Probability of Receiving Message y Given State z (P(y/z)) | |
|---|---|---|
| | Test Passed ($y_1$) | Test Failed ($y_2$) |
| Working ($z_1$) | .9 | .1 |
| Not working ($z_2$) | .3 | .7 |

Table 5

PROPERTIES OF TEST $Y_3$

(Static Firing Test of Propulsion Subsystem)

| State of Subsystem with respect to System X Requirements | Probability of Receiving Message y Given State z (P(y/z)) | |
|---|---|---|
| | Test Passed ($y_1$) | Test Failed ($y_2$) |
| Working ($z_1$) | .95 | .05 |
| Not working ($z_2$) | .45 | .55 |

Table 6

PROPERTIES OF TEST $Y_{1,2}$
(Centrifuge Test and Sled Test of Guidance Subsystem)

| State of Subsystem with respect to System X Requirements | Probability of Receiving Message y Given State z (P(y/z)) | | | | | |
|---|---|---|---|---|---|---|
| | Centr: $y_1$ Sled: $y_1$ ($y_{1,1}$) | Centr: $y_1$ Sled: $y_2$ ($y_{1,2}$) | Centr: $y_2$ Sled: $y_1$ ($y_{2,1}$) | Centr: $y_2$ Sled: $y_2$ ($y_{2,2}$) | Centr: $y_3$ Sled: $y_1$ ($y_{3,1}$) | Centr: $y_3$ Sled: $y_2$ ($y_{3,2}$) |
| Working ($z_1$) | .72 | .08 | .18 | .02 | 0 | 0 |
| Not working ($z_2$) | .06 | .14 | .12 | .28 | .12 | .28 |

Table 7

COST OF AVAILABLE REDESIGN ACTIONS AS A FUNCTION OF MESSAGES RECEIVED:
TESTS $Y_1$, $Y_2$, AND $Y_3$

| Action Taken on Conclusion of Test | Cost of Redesign Action ($c_Y(y, a)$) | | | | | | |
|---|---|---|---|---|---|---|---|
| | Test $Y_1$ | | | Test $Y_2$ | | Test $Y_3$ | |
| | Message | | | Message | | Message | |
| | $y_1$ | $y_2$ | $y_3$ | $y_1$ | $y_2$ | $y_1$ | $y_2$ |
| $a_0$ None | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $a_1$ Redesign to obtain $y_1$ | -- | 4 | 8 | -- | 5 | -- | 6 |
| $a_2$ Redesign to obtain $y_2$ | -- | -- | 3 | -- | -- | -- | -- |

Table 8

COST OF AVAILABLE REDESIGN ACTIONS AS A FUNCTION OF MESSAGES RECEIVED: TEST $Y_{1,2}$

| | Cost of Redesign Action ($c_Y(y, a)$) | | | | | |
|---|---|---|---|---|---|---|
| | Message Received | | | | | |
| Action Taken on Conclusion of Test | $y_{1,1}$ | $y_{1,2}$ | $y_{2,1}$ | $y_{2,2}$ | $y_{3,1}$ | $y_{3,2}$ |
| $a_0$ None | 0 | 0 | 0 | 0 | 0 | 0 |
| $a_{1,1}$ Redesign to obtain $y_{1,1}$ | -- | 5 | 4 | 7 | 8 | 9 |
| $a_{2,1}$ Redesign to obtain $y_{2,1}$ | -- | -- | -- | -- | 3 | 6 |

Table 9

AVAILABLE REDESIGN ACTIONS: TESTS $Y_{1,3}$, $Y_{2,3}$, AND $Y_{1,2,3}$

| Test $Y_{1,3}$ | | Test $Y_{2,3}$ | | Test $Y_{1,2,3}$ | |
|---|---|---|---|---|---|
| Message | Available Actions | Message | Available Actions | Message | Available Actions |
| $y_{1,1}$ | $a_0$ | $y_{1,1}$ | $a_0$ | $y_{1,1,1}$ | $a_0$ |
| $y_{1,2}$ | $a_0$, $a_{1,1}$ | $y_{1,2}$ | $a_0$, $a_{1,1}$ | $y_{1,1,2}$ | $a_0$, $a_{1,1,1}$ |
| $y_{2,1}$ | $a_0$, $a_{1,1}$ | $y_{2,1}$ | $a_0$, $a_{1,1}$ | $y_{1,2,1}$ | $a_0$, $a_{1,1,1}$ |
| $y_{2,2}$ | $a_0$, $a_{1,1}$, $a_{1,2}$, $a_{2,1}$ | $y_{2,2}$ | $a_0$, $a_{1,1}$, $a_{1,2}$, $a_{2,1}$ | $y_{1,2,2}$ | $a_0$, $a_{1,1,1}$, $a_{1,1,2}$, $a_{1,2,1}$ |
| | | | | $y_{2,1,1}$ | $a_0$, $a_{1,1,1}$ |
| $y_{3,1}$ | $a_0$, $a_{1,1}$, $a_{2,1}$ | | | $y_{2,1,2}$ | $a_0$, $a_{1,1,1}$, $a_{1,1,2}$, $a_{2,1,1}$ |
| $y_{3,2}$ | $a_0$, $a_{1,1}$, $a_{1,2}$, $a_{2,1}$, | | | $y_{2,2,1}$ | $a_0$, $a_{1,1,1}$ |
| | $a_{2,2}$, $a_{3,1}$ | | | $y_{2,2,2}$ | $a_0$, $a_{1,1,1}$, $a_{1,1,2}$, $a_{2,2,1}$ |
| | | | | $y_{3,1,1}$ | $a_0$, $a_{1,1,1}$, $a_{2,1,1}$ |
| | | | | $y_{3,1,2}$ | $a_0$, $a_{1,1,1}$, $a_{1,1,2}$, $a_{2,1,1}$, $a_{2,1,2}$, $a_{3,1,1}$ |
| | | | | $y_{3,2,1}$ | $a_0$, $a_{1,1,1}$, $a_{2,1,1}$ |
| | | | | $y_{3,2,2}$ | $a_0$, $a_{1,1,1}$, $a_{1,1,2}$, $a_{2,1,1}$, $a_{2,1,2}$, $a_{3,2,1}$ |

The costs of the redesign actions given in Table 9 are obtained from Tables 7 and 8 by simple addition since $Y_1$ and $Y_3$, $Y_2$ and $Y_3$, and $Y_{1,2}$ and $Y_3$ apply to different subsystems. Thus, for example,

$$c_{Y_{1,2,3}}(y_{3,2,2}, a_{2,1,1}) = c_{Y_{1,2}}(y_{3,2}, a_{2,1}) + c_{Y_3}(y_2, a_1)$$
$$= 6 + 6 = 12.$$

From $B_i = E_i + L_i + R_i'$, we obtain that $B_1 = 15$ and that $B_2 = 15$. Applying Theorem 2 to (1), we find by reference to (10) that the minimum expected final checkout cost of system X at the beginning of the final test stage, $f_1(p_1, p_2)$, is

$$(24) \qquad f_1(p_1, p_2) = 61 - 16p_1 - 14p_2 - 15p_1p_2 + \min\{-3p_2 - 12, -5p_1 - 10\}.$$

For example, when $p_1$ (the probability as of the beginning of the final test stage that the guidance subsystem will work) is .5, and $p_2$ (the corresponding probability for the propulsion subsystem) is .5, we obtain

$$f_1(.5, .5) = 28.75, \qquad S^* = (1, 2),$$

and when $p_1 = .9$ and $p_2 = .7$,

$$f_1(.9, .7) = 12.85, \qquad S^* = (2, 1).$$

Thus, in case the first test firing of the missile fails, the optimal sequence in the first case is to perfect the guidance subsystem before attempting another firing of the full system and, if necessary, proceed

to redesign the propulsion subsystem. In the second case, the reverse sequence should be used.

The total minimum expected cost of testing and redesign for each of the possible tests that may be used in stage 1, as of the beginning of that stage (assuming an optimal test sequence in stage 2), is shown in Table 10 for selected probability vectors $\bar{p}$. When $\bar{p} = (.5, .5)$, we note that test $Y_{1,2,3}$ gives the lowest expected cost: $f_2(.5, .5) = 20.63$. Analogously, we note that $f_2(.5, .95) = 11.06$ ($Y_{1,2}$ optimal), $f_2(.9, .7) = 10.42$ ($Y_{1,3}$ optimal), and $f_2(.95, .95) = 3.61$ ($Y_1$ optimal). It should be noted from Table 10 that the null test ($Y_2$) is the worst test when $\bar{p} = (.5, .5)$, while it is the second best test when $\bar{p} = (.95, .95)$.

The best corrective actions $\alpha^*_{Y,\bar{p}}(y)$ for the best tests in Table 10 are given in Table 11. Results for tests $Y_1$ and $Y_{1,3}$ illustrate that it is not always optimal to redesign so as to pass a given test on all counts when the outcome of the first test trial is unfavorable.

Let us illustrate briefly how $f_2(.9, .7) = 10.42$ was computed for test $Y_{1,3}$.

<u>Step 1</u>. Bayes' Theorem gives, by direct application of the conditional probabilities in Tables 3 and 5,

$$\bar{p}'(y_{1,1}, \bar{p}) = \left(\frac{.8p_1}{.8p_1 + .2(1 - p_1)}, \frac{.95p_2}{.95p_2 + .45(1 - p_2)}\right),$$

so that $\bar{p}'(y_{1,1}, (.9, .7)) = (.97, .83)$. Inserting this value in (24) we obtain $f_1(.97, .83) = 6.93$. In the same way, we obtain for the

Table 10

TOTAL MINIMUM EXPECTED COST OF TESTING AND REDESIGN OF SYSTEM X, AS OF THE BEGINNING OF THE FIRST TEST STAGE, AS A FUNCTION OF THE TEST USED AT THE FIRST STAGE FOR SELECTED PROBABILITY VECTORS

| *A Priori* Probability Vector | | Test Used at First Stage | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $P_1$ | $P_2$ | $Y_0$ | $Y_1$ | $Y_2$ | $Y_3$ | $Y_{1,2}$ | $Y_{1,3}$ | $Y_{2,3}$ | $Y_{1,2,3}$ |
| .5 | .5 | 28.95 | 24.10 | 26.62 | 27.14 | 24.32 | 22.18 | 34.59 | <u>20.63</u> |
| .5 | .95 | 17.72 | 11.55 | 14.16 | 19.79 | <u>11.06</u> | 12.18 | 15.73 | 11.62 |
| .9 | .7 | 12.85 | 11.93 | 13.76 | 11.47 | 13.61 | <u>10.42</u> | 12.27 | 12.26 |
| .95 | .95 | 4.07 | <u>3.61</u> | 5.78 | 4.59 | 5.37 | 4.10 | 6.32 | 5.74 |

Table 11

BEST CORRECTIVE ACTION (a) AS A FUNCTION OF THE MESSAGE RECEIVED (y) FOR SELECTED FIRST-STAGE TESTS AND PROBABILITY VECTORS

| Test $Y_1$ $(\bar{p} = (.95, .95))$ | | Test $Y_{1,2}$ $(\bar{p} = (.5, .95))$ | | Test $Y_{1,3}$ $(\bar{p} = (.9, .7))$ | | Test $Y_{1,2,3}$ $(\bar{p} = (.5, .5))$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| $y_1$ | $a_0$ | $y_{1,1}$ | $a_0$ | $y_{1,1}$ | $a_0$ | $y_{1,1,1}$ | $a_0$ | $y_{2,2,1}$ | $a_{1,1,1}$ |
| $y_2$ | $a_0$ | $y_{1,2}$ | $a_{1,1}$ | $y_{1,2}$ | $a_{1,1}$ | $y_{1,1,2}$ | $a_{1,1,1}$ | $y_{2,2,2}$ | $a_{1,1,1}$ |
| $y_3$ | $a_2$ | $y_{2,1}$ | $a_{1,1}$ | $y_{2,1}$ | $a_{1,1}$ | $y_{1,2,1}$ | $a_{1,1,1}$ | $y_{3,1,1}$ | $a_{1,1,1}$ |
| | | $y_{2,2}$ | $a_{1,1}$ | $y_{2,2}$ | $a_{1,1}$ | $y_{1,2,2}$ | $a_{1,1,1}$ | $y_{3,1,2}$ | $a_{1,1,1}$ |
| | | $y_{3,1}$ | $a_{1,1}$ | $y_{3,1}$ | $a_{2,1}$ | $y_{2,1,1}$ | $a_{1,1,1}$ | $y_{3,2,1}$ | $a_{1,1,1}$ |
| | | $y_{3,2}$ | $a_{1,1}$ | $y_{3,2}$ | $a_{2,1}$ | $y_{2,1,2}$ | $a_{1,1,1}$ | $y_{3,2,2}$ | $a_{1,1,1}$ |

other possible messages:

$$\bar{p}'(y_{1,2}, (.9, .7)) = (.97, .18) \qquad f_1(.97, .18) = 25.49$$

$$\bar{p}'(y_{2,1}, (.9, .7)) = (.82, .83) \qquad f_1(.82, .83) = 11.56$$

$$\bar{p}'(y_{2,2}, (.9, .7)) = (.82, .18) \qquad f_1(.82, .18) = 29.05$$

$$\bar{p}'(y_{3,1}, (.9, .7)) = (0, .83) \qquad f_1(0, .83) = \infty^{\dagger}$$

$$\bar{p}'(y_{3,2}, (.9, .7)) = (0, .18) \qquad f_1(0, .18) = \infty^{\dagger}$$

Step 2. We must next compute

$$F_{Y_{1,3}}(y, \bar{p}) \equiv \min_{a \in A_{Y,y}} \{c_{Y_{1,3}}(y, a, \bar{p}) + f_1(\bar{p}'(y'(a, \bar{p})))\}.$$

In the case of message $y_{2,2}$, for example, we note from Table 9 that

$$A_{Y_{1,3}, y_{2,2}} = \{a_0, a_{1,1}, a_{1,2}, a_{2,1}\}.$$

From Table 7 we obtain

$$c_{Y_{1,3}}(y_{2,2}, a_0, \bar{p}) = 0,$$

$$c_{Y_{1,3}}(y_{2,2}, a_{1,1}, \bar{p}) = 4 + 6 = 10,$$

$$c_{Y_{1,3}}(y_{2,2}, a_{1,2}, \bar{p}) = 4,$$

$$c_{Y_{1,3}}(y_{2,2}, a_{2,1}, \bar{p}) = 6.$$

---

† When some element of $\bar{p}'$ is 0, $f_N(\bar{p}')$ is automatically set equal to infinity in order to rule out the null action. See Assumption 3.

Thus,

$$F_{Y_{1,3}}(y_{2,2}, (.9, .7))$$

$$= \min \{0 + 29.05,\ 10 + 6.93,\ 4 + 25.49,\ 6 + 11.56\} = 16.[illegible]$$

Consequently, action $a_{1,1}$ is optimal when message $y_{2,2}$ is received. From the formula $P(y) = \Sigma_z P(y/z)P(z)$, we obtain, by reference to Tables 3 and 5 $P(y_{2,2}) = (.2 \times .05)(.9 \times .7) + (.2 \times .55)(.9 \times .3) + (.4 \times .05)(.1 \times .7) + (.4 \times .55)(.1 \times .3) = .04$. Repeating the procedure for each message, we obtain:

| | $F_{Y_{1,3}}(y, (.9, .7))$ | $\alpha^*_{Y_{1,3}, (.9, .7)}(y)$ | $P(y)$ |
|---|---|---|---|
| $y_{1,1}$ | 6.93 | $a_0$ | .59 |
| $y_{1,2}$ | 12.93 | $a_{1,1}$ | .15 |
| $y_{2,1}$ | 10.93 | $a_{1,1}$ | .18 |
| $y_{2,2}$ | 16.93 | $a_{1,1}$ | .04 |
| $y_{3,1}$ | 14.56 | $a_{2,1}$ | .03 |
| $y_{3,2}$ | 20.56 | $a_{2,1}$ | .01 |

Note that the first and third columns appear in Table 11.

<u>Step 3</u>. Since $T(Y_{1,3}) = 1.1$ (Table 2), we obtain from the preceding summary that

$$T(Y_{1,3}) + \sum_{y \in Y_{1,3}} P(y) F_{Y_{1,3}}(y, (.9, .7)) = 1.1 + 9.32 = 10.42.$$

Since this is the minimum value given by

$$\min_{Y \in \mathcal{Y}} \left\{ T(Y) + \sum_{y \in Y} P(y) F_Y(y, (.9, .7)) \right\},$$

$f_2(.9, .7) = 10.42.$

## IV. THE FINAL CHECKOUT PROBLEM WHEN TEST RESULTS ARE MULTIVALUED

In the preceding model, we limited the discussion to the case in which the final checkout information was two-valued. This restriction, however, has not been present with respect to the pre-checkout tests of parts, since the model is the same whether y is such that it can take only two values or whether it may assume several values.

Multivalued test results correspond to the situation in which one learns more about the object tested than merely whether it passes or fails the test. Thus, in putting a system through a final test, one may often ascertain whether specific subsystems work even though the system itself fails the test.

### MULTIPLE CHOICE OF TESTS

Consider a system of K components in which, through test configuration Y, one obtains $k + 1$ read-outs, where $k \leq K'$. These correspond to $k + 1$ messages, one representing the system as a whole, and the others k different components. Let the indices of these components form the set $K_1$. Thus, we may write $y = (y_0, y_1, y_2, \ldots, y_k)$, where each $y_i$ is two-valued since the final checkout always gives perfect information. Thus $2^{k+1}$ different messages may be distinguished when $k < K'$, and $2^k$ when $k = K'$. Let S be such that the components about which specific information is received are at the beginning of S. Letting $E_Y$ be the cost of the final test when test combination Y is used, the cost of the final checkout (not including the initial system test), $C_Y(S)$, becomes when this sequence is used:

$$C_Y(S) = \pi_1 0 + (1 - p_1)(R_1 + R_1') + C_1^* + \cdots + (1 - p_k)(R_k + R_k')$$
$$+ C_k^* + (1 - p_1, \ldots, p_k)E_y + (1 - \pi_{k+1})B_{k+1} + C_{k+1}^*$$
$$+ q_{k+1}E_Y + \ldots + (1 - \pi_{K'})B_{K'} + C_{K'}^* + q_{K'}E_Y - q_{K'}E_{YK'},$$

where the last term is zero when $k = K'$. Simplifying,

$$(25) \qquad C_Y(S) = \sum_{k\epsilon K_1} \left(q_k(R_k + R_k') + C_k^*\right) + (1 - p_1, \ldots, p_k)E_Y$$
$$+ \sum_{k\notin K_1} (B_k + C_k^* + q_kE_Y)$$
$$- \sum_{k\notin K_1} \pi_kB_k - \begin{cases} q_{K'}E_{YK'} & \text{when } K_1 \neq \varphi, \\ 0 & \text{when } K_1 = \varphi. \end{cases}$$

After test $E_Y$ is performed, we first redesign those components which the test shows are not working. With the remaining components, we proceed as in Fig. 3 until $y_o$ indicates that the system is working. It is clearly optimal to redesign the components about which specific information is received before reworking any others. To see this, let component $j \epsilon K_1$ be transferred to the position following $i \notin K_1$. This has the sole effect of changing $(1 - \pi_m)B_m$ to $(1 - p_j\pi_m)B_m$, $m = k + 1$, $\ldots$, $i$, and $(1 - p_1, \ldots, p_k)E_Y$ to $(1 - p_1, \ldots, p_{j-1}p_{j+1}, \ldots, p_k)E_Y + (1 - p_j)E_Y$, all of which increase $C_Y$. This gives Theorem 6.

Theorem 6. Let each final test be such that it provides information as to whether the following work: (1) The system, and (2) each component $k \epsilon K_1$. Then, after the initial system test, the optimal sequence is to redesign, as necessary, all components $k \epsilon K_1$, in any order. At this point, proceed as in Theorem 4 for all components $k \notin K_1$.

Thus the minimum expected cost of the final checkout becomes, by (25) and (10),

$$f_1(\bar{p}) = \min_{Y_1 \in \mathcal{Y}_1} \{E_{Y_1} + C^*_{Y_1}(\bar{p})\}. \tag{26}$$

SERIAL DEPENDENCE AMONG MESSAGES

We now introduce a further complication with respect to the final checkout of a system. The situation to be discussed is one in which multivalued messages are received due to the fact that subsystems function serially. That is, a particular subsystem may be designed to go into operation only after other subsystems have begun, or even completed, their functions. Thus, if the "lead" subsystem fails, the test clearly cannot provide information with respect to a "successor" subsystem. We refer to "no message" as the null message.

An example of this kind of system is given by a multistage missile. Unless the first-stage rocket operates, we will not have an opportunity to learn whether the second or third stages, for example, work.

Definition 1. A subsystem belongs to the first stage of a system operation[†] if its activation is not dependent on whether some other subsystem works.

This definition is readily extended by induction to

Definition 2. A subsystem belongs to the nth stage of a system operation, n = 2, 3, ..., if its activation is dependent on whether a subsystem in the n - 1th stage works and the subsystem does not already belong to a previous stage.

---

[†]The various stages of system operation should not be confused with the stages (phases) of testing. Unfortunately, the word "stage" is the most appropriate in both instances.

Let L be the total number of operational stages obtained by this procedure and let $K_j'$ be the set of subsystem indices belonging to the jth operational stage, $j = 1, \ldots, L$, for which $p_i < 1$. Clearly, $L \leq K'$, and $\cup_{j=1}^{L} K_j' = \{1, \ldots, K'\}$; that is, each subsystem is identified with one and only one operational stage. In each stage, denote by $K_j \subseteq K_j'$ the set of subsystems about which test Y provides specific information. Now let the test sequence S be as follows: Let all subsystems of a stage be adjacent to each other, with the subsystems about which specific information is obtained by the test in front of the others. Then position the operational stages in ascending order. We then obtain

$$(27) \qquad C_Y(S) = \sum_{j=1}^{L} \Big[ \sum_{i \in K_j} \Big( q_i(R_i + R_i') + C_i^* \Big) + (1 - \prod_{i \in K_j} p_i) E_Y$$
$$+ \sum_{i \in K_j' - K_j} (B_i + C_i^* + q_i E_Y - R_{ij} B_i) - q_{s_j} E_{Y s_j} \Big],$$

where $R_{ij} = \Pi_{k=1}^{K_j'} p_k$, and $s_j$ is the last subsystem in the jth stage. Within each stage, the situation is clearly completely analogous to the one-stage case so that Theorem 6 applies directly.

Now suppose we change S by moving subsystem $j \in K_m'$ to the position behind subsystem $i \in K_n'$, where $m < n$. Our test and redesign operations would then clearly be trapped in the mth operational stage if the transferred subsystem does not work, since in that case we would not be able to obtain any information about subsequent stages. Since $q_j > 0$ for all j involved by Theorem 3, there is a positive probability that this will happen. Thus we obtain

Theorem 7. Given a system that operates in L stages and a final test that provides information as to whether the system and each subsystem $k \in K_j$, $j = 1, \ldots, L$, work, the optimal test sequence, $S^*$, is obtained as follows: (1) Group the subsystems by stage and order the stages by their natural number; (2) within each stage, order the subsystems as per Theorem 6.

## V. PARALLEL DEVELOPMENT AS A SPECIAL CASE

An interesting attempt to model the phenomenon of parallel R&D efforts has been made by Nelson [7]. We now demonstrate how that model may be viewed as a special case of the model developed in this study.

Nelson considers the problem of developing a system to meet certain specifications at minimum cost in time and money. He distinguishes between two stages: design and development completion. In the design stage, opportunities for learning exist that may be beneficially utilized in the second stage. These opportunities are in the form of a "built-in" test which gives two-valued perfect information at the end of the first stage for a given design.

The following set of strategies is considered by Nelson: The developer has the opportunity to run independent, but merit-wise indistinguishable, projects in the first stage, each costing the same amount of money. (As before, we ignore time considerations.) Then, at the end of the first stage, he is to pick one project to run to completion. We denote a success in the first-stage test of a <u>design</u> by $y_1$ and failure by $y_2$. For any number of designs i, there is clearly no benefit in obtaining more than one $y_1$ since only one design is to be pursued further. Thus there are only two macromessages that are relevant for a given i; let $y_{i1}$ be the set of messages containing at least one $y_1$ among the i designs tested and let $y_{i2}$ be the message that $y = y_2$ for all i. Let us also make the following definitions:

M = average cost of a design incurred in the first stage, including testing

p = probability of obtaining message $y_2$ for a design

$M_1$ = total test and redesign cost in second stage when $y_{i1}$ is obtained

$M_2$ = total test and redesign cost in second stage when $y_{i2}$ is obtained

$K$ = total development cost in second stage exclusive of testing and redesign

When $i$ designs are pursued in the first stage, the total expected development cost of the system becomes

$$M + (i - 1)M + K + (1 - p^i)M_1 + p^i M_2. \tag{28}$$

Thus, since M and K are fixed, the minimum expected cost of testing and redesign is

$$\min_{i \in I} \{(i - 1)M + (1 - p^i)M_1 + p^i M_2\}, \tag{29}$$

where I is the set of positive integers.

Comparing (29) with (22), we find that

$$\mathcal{Y} = I,$$

$$Y_i = i,$$

$$T(Y_i) = (i - 1)M,$$

$$P(y_{i1}) = 1 - p^i,$$

$$P(y_{i2}) = p^i,$$

$$c_Y^*(y, a, \bar{p}) = 0,$$

$$y' = y_{ij},$$

$$C^*(\bar{p}'(\bar{p}, y_{ij})) = M_j,$$

so that the preceding model of parallel development may be viewed as a special case of the testing problem considered in this study.

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**10. ABSTRACT**

Experience indicates that the major source of uncertainty in weapon system development can be traced to activities involving testing and redesign, yet surprisingly little of a conceptual nature has been done to improve the decisionmaking process involved in performing these activities. An adaptive model of the testing process is constructed that is designed to provide the project director and his staff with a means for determining the best test to perform at a given stage in the development of a system and to enable the same decisionmakers to choose intelligently among the available redesign actions once the test results are known. Although the model is presented in terms of relatively simple systems and tests, it should be capable of handling those of a highly complex nature.

**11. KEY WORDS**

Testing
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